

Introduction

- Image segmentation is inherently ambiguous;
- Modelling this ambiguity enables predicting **multiple plausible segmentations for the same image**;
- Applications not only in medical imaging but also in computer vision.

Even human experts disagree on the shape of different lung nodules:

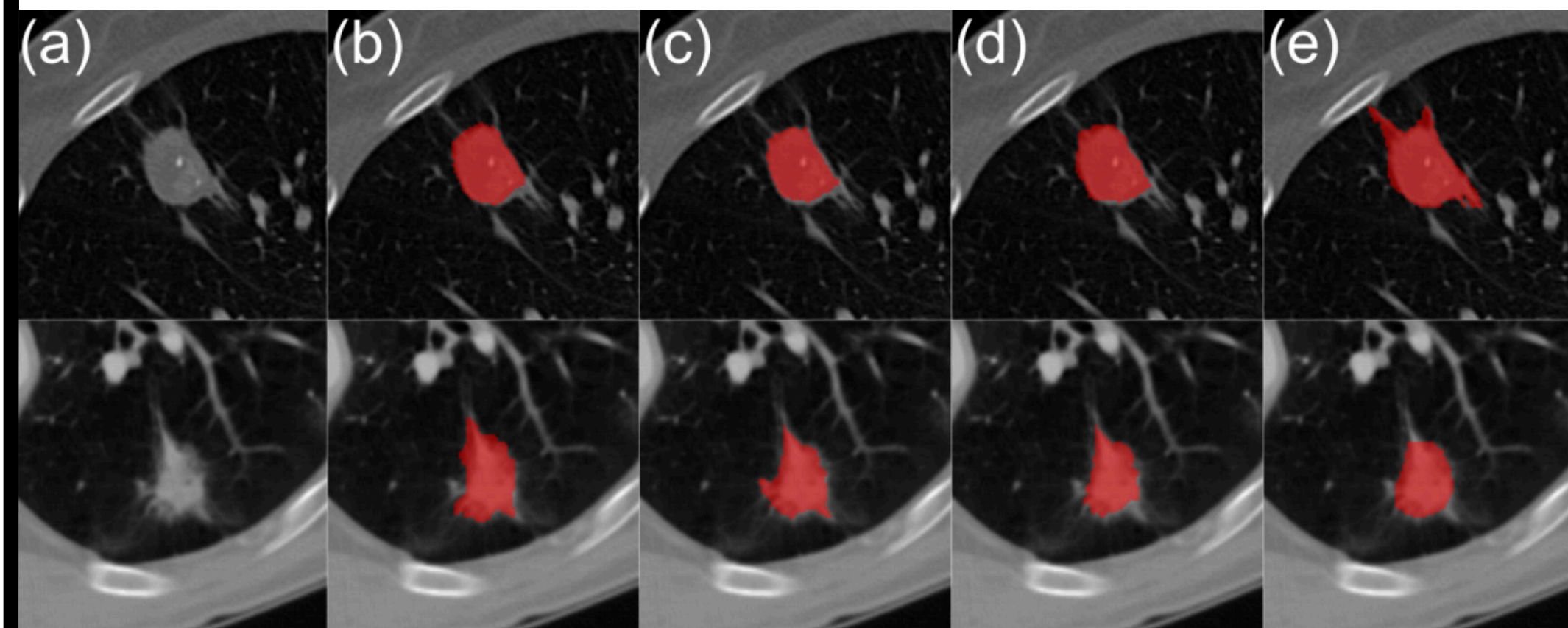


Image (a), ground-truth segmentation from four different human experts (b-e).

Problem: Sampling from the posteriors of a standard neural network produces spatially incoherent results because the pixels are treated independently:

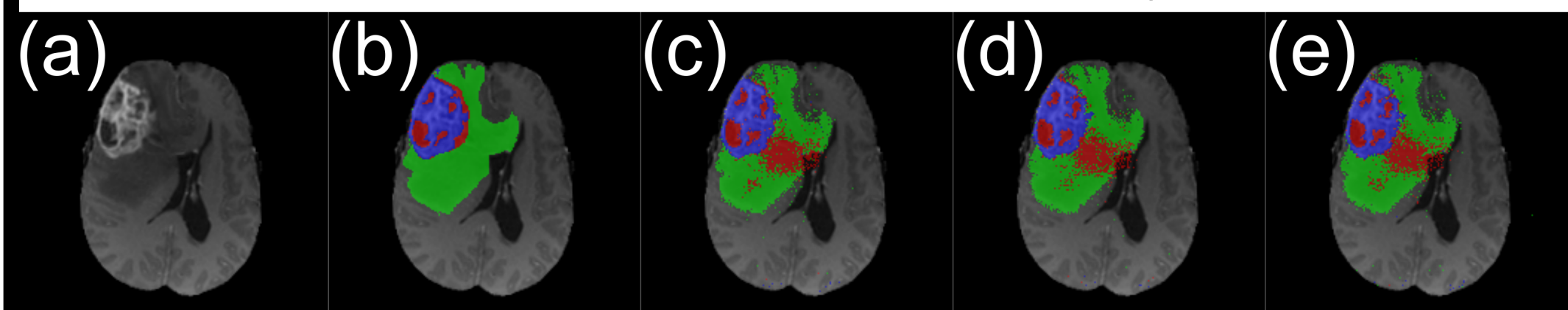


Image (a), ground-truth segmentation (b), three random samples (c-e). Note these are 2D slices of 3D images.

Methods

Solution: To make the logits dependent of each other by modelling them with a low-rank multivariate normal distribution:

$$p(\boldsymbol{\eta} | \mathbf{x}) = \mathcal{N}(\boldsymbol{\eta} | \boldsymbol{\mu}(\mathbf{x}), \boldsymbol{\Sigma}(\mathbf{x})),$$

where \mathbf{x} is the image and $\boldsymbol{\eta}$ the logit-map.

- The non-diagonal covariance matrix, $\boldsymbol{\Sigma}$, models dependencies between pixels;
- Its low-rank parametrisation, $\boldsymbol{\Sigma} = \mathbf{P}\mathbf{P}^T + \mathbf{D}$, enables the distribution to be efficiently computed by a neural network even for large 3D images;
- The overhead is only predicting the three maps that describe the distribution, instead of one, at the output of the network;
- The rank of the covariance matrix can be adjusted to control the expressiveness of the distribution;
- The new loss function can be obtained using Monte-Carlo integration:

$$-\log p(\mathbf{y} | \mathbf{x}) \approx -\log \frac{1}{M} \sum_{m=1}^M p(\mathbf{y} | \boldsymbol{\eta}^{(m)}),$$

$$\boldsymbol{\eta}^{(m)} | \mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}(\mathbf{x}), \boldsymbol{\Sigma}(\mathbf{x})).$$

Results

Multiple spatially consistent samples for the same image:

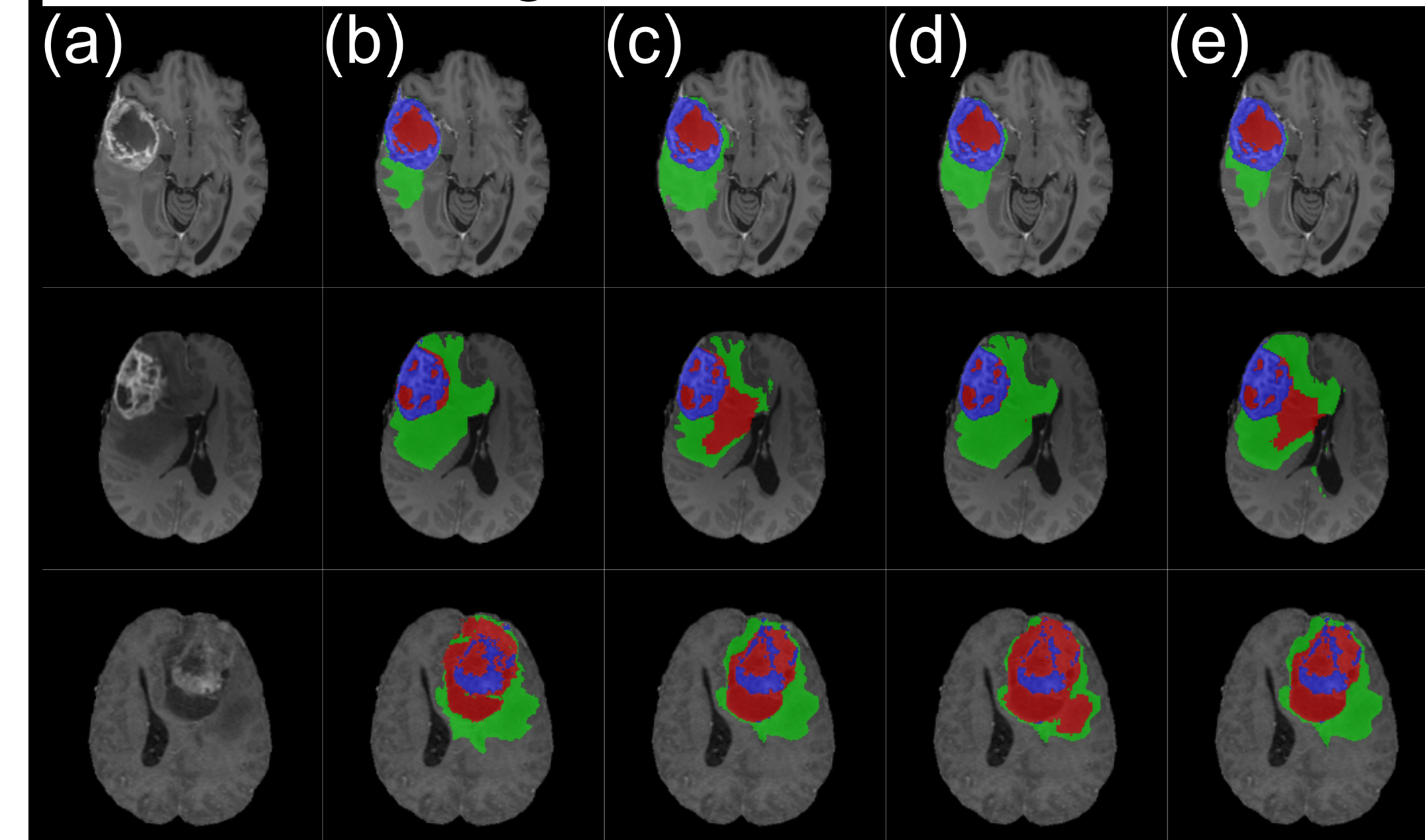


Image (a), ground-truth segmentation (b), three random samples (c-e). Note these are 2D slices of 3D images.

Comparison with SOTA:

- SOTA algorithms are based on conditional VAEs and strict, memory-intensive architectures;
- In contrast, our approach is light-weight and flexible enabling it to be used over any neural network architecture;
- We can generate infinitely many samples from one forward pass;
- In lung nodule segmentation, we obtain equal or better results in predictive performance and distance to the human expert distribution.